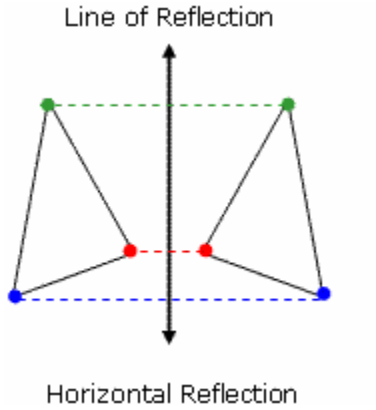
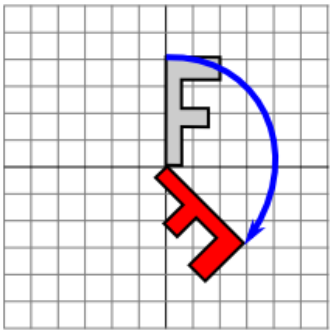
Transformation

Class Notes – IGCSE Mathematics 0580

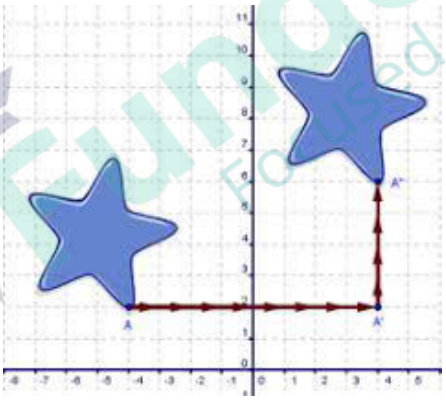


- Reflection
- Rotation
- Enlargement
- Translation

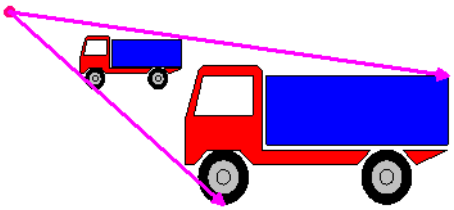
Rotation



Translation



Enlargement



By Mariam Javaid Amir

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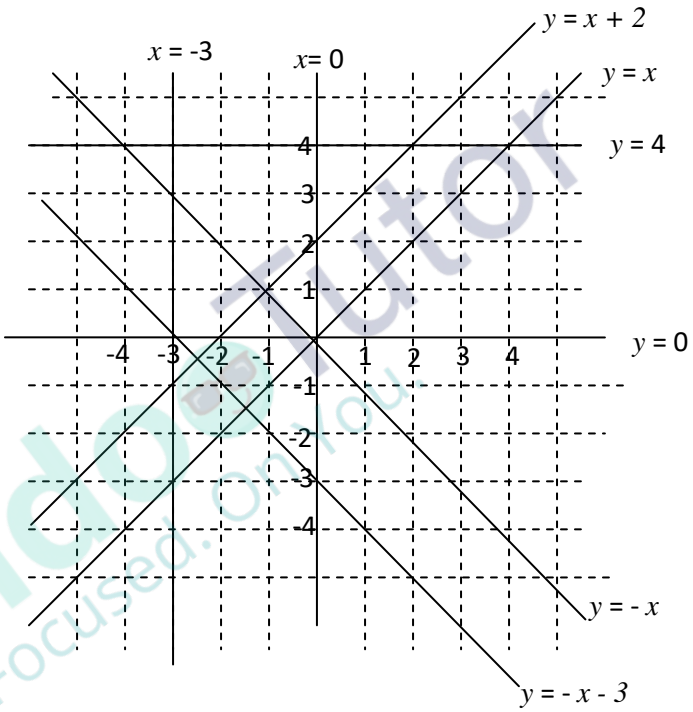
Transformation -: Changing a set of coordinates into another set of coordinates. Example (1,0) changing into (0,1) by applying a transformation onto it.

The four types of transformations
1) Reflection
2) Rotation
3)Enlargement
4)Translation

1) Reflection (M) :-

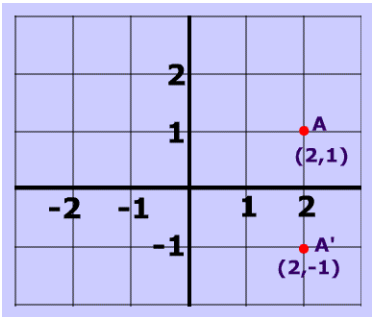
A reflection is a 'flip' of a shape over the line of reflection.

Some equation of straight lines are shown below (revision)



a) Reflection on horizontal line :- Note the vertical distance between the object and horizontal line of reflection and move the object the same distance in the opposite direction.

Example the figure shows reflection of object A on line $y = 0$ (x axis) into image A'

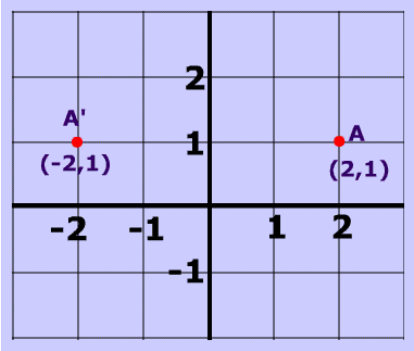


The general rule for a reflection in x axis
(A,B) → (A, - B)

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b) Reflection on vertical line :- Note the horizontal distance between the object and vertical line of reflection and move the object the same distance in the opposite direction.

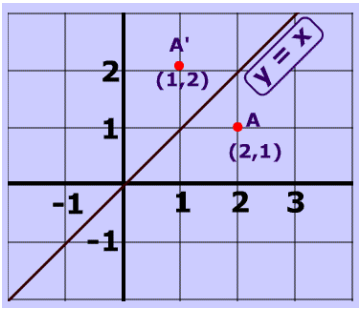
Example the figure shows reflection of object A on line $x = 0$ (y axis) into image A'



The general rule for a reflection in x axis
(A,B) → (-A, B)

c) Reflection on tilted line :- :- Note the diagonal distance between the object and the tilted line of reflection and move the object the same distance in the opposite direction.

Example the figure shows reflection of object A on line $y = x$ into image A'



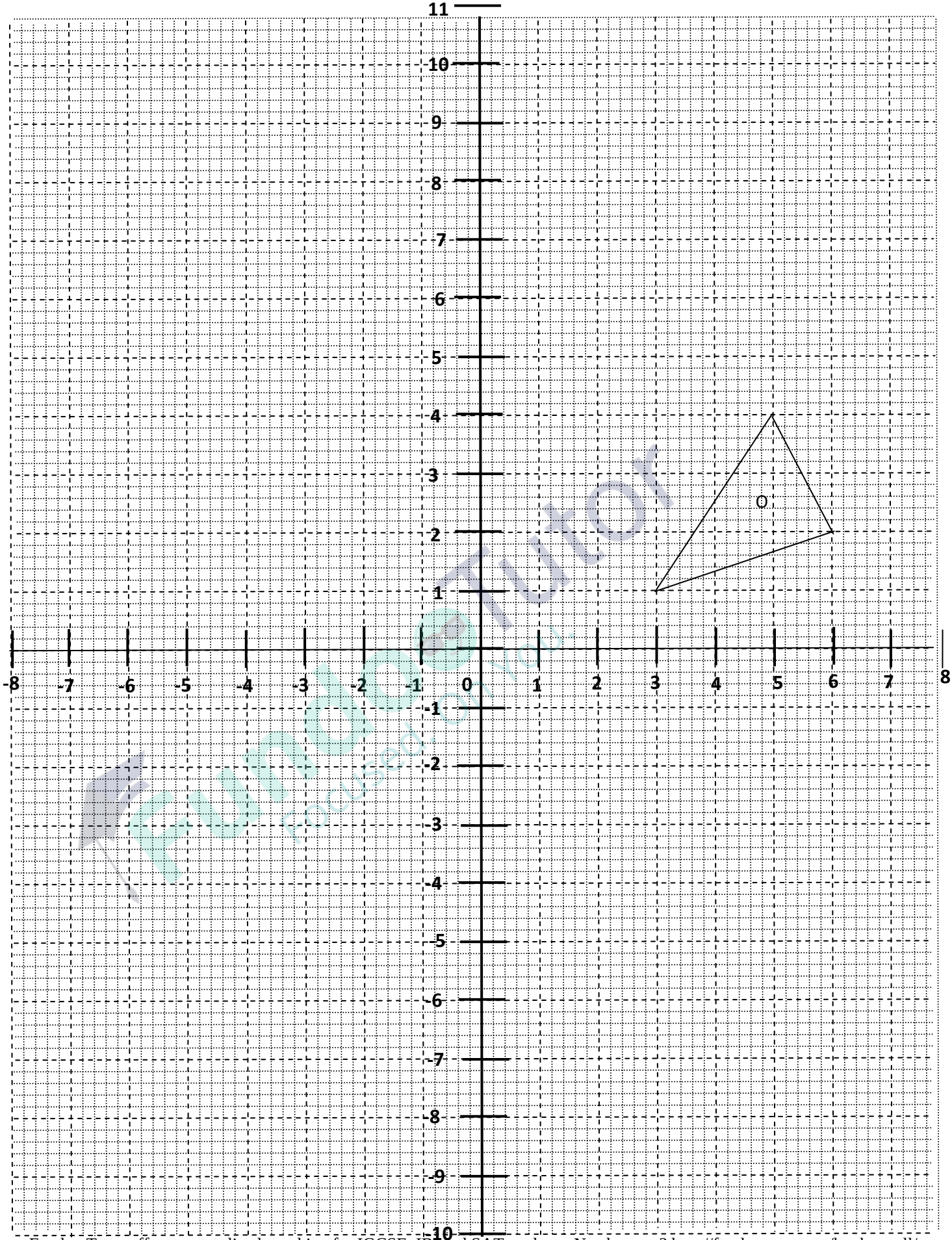
The general rule for a reflection in $y = x$
(A,B) → (B, A) similarly,

The general rule for a reflection in $y = -x$
(A,B) → (-B, -A)

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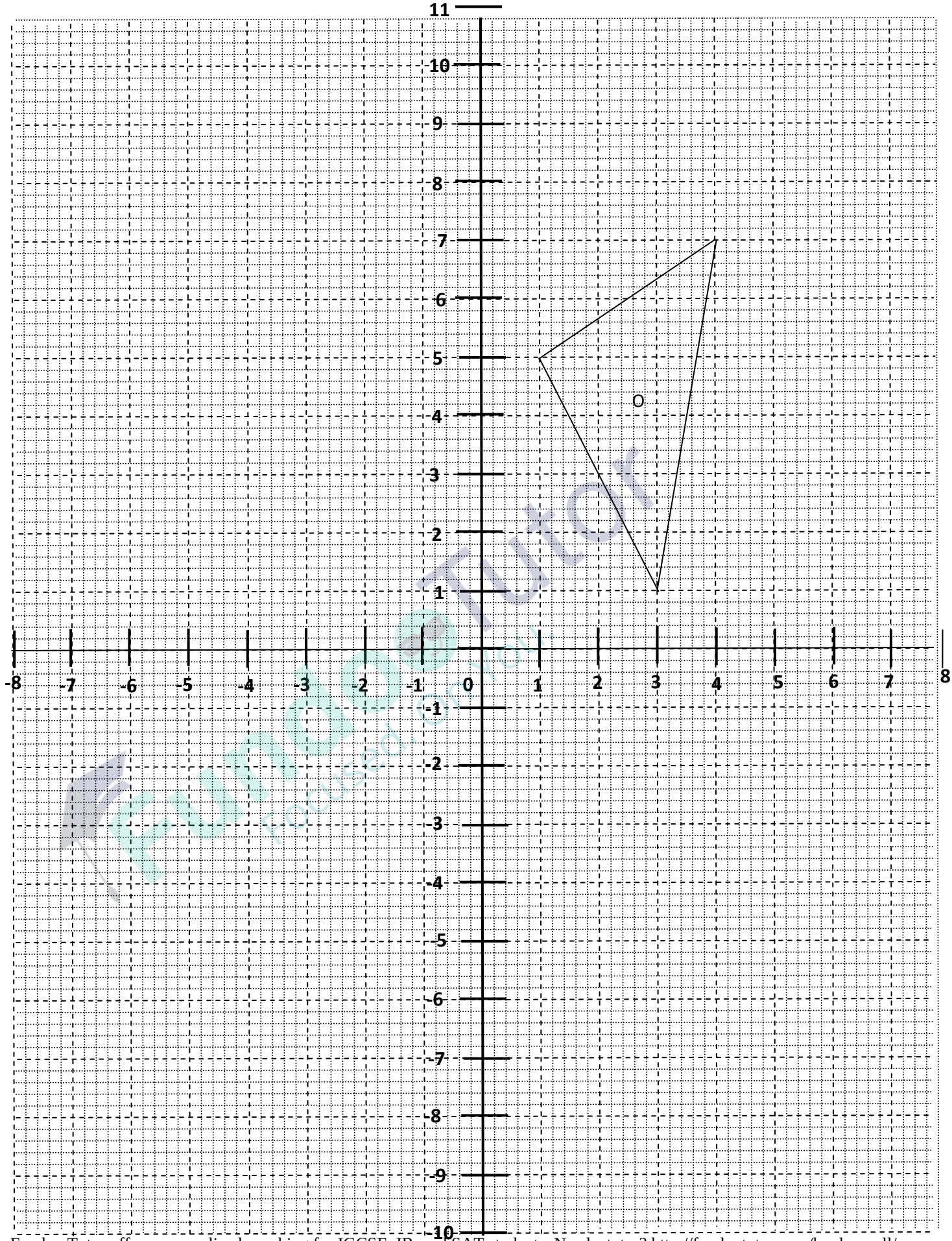
Q1) Reflect triangle O in the line **a)** $y = 0$ **b)** $x = 0$ **c)** $y = x$ **d)** $y = -x$



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Ans a) (3,-1) (6,-2) (5,-4) b) (-3,1) (-6,2)(-5,4) c) (1,3) (4,5) (2,6) d) (-1,-3) (-4,-5) (-2,-6)

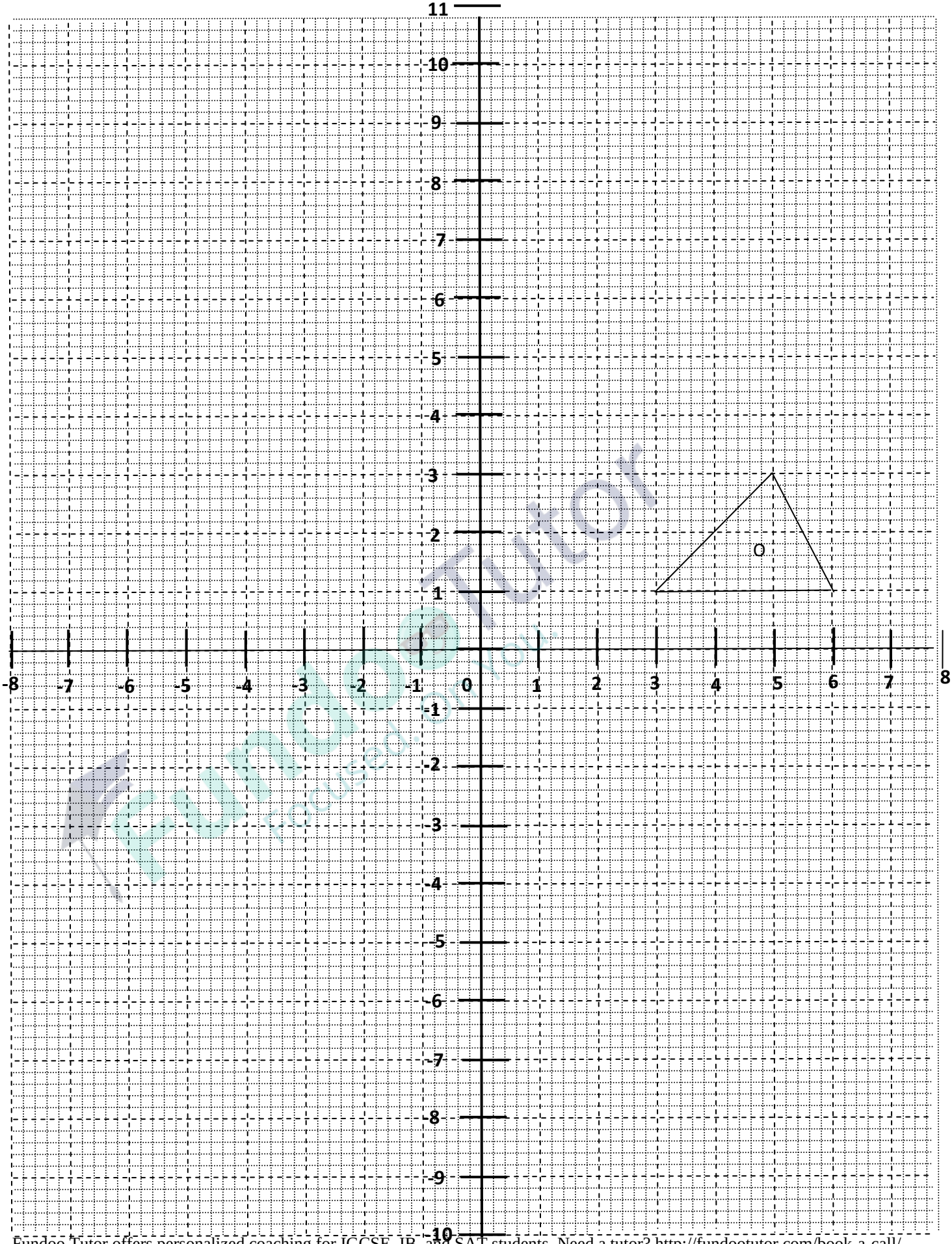
Q2) Reflect triangle O in the line a) $y = 0$ b) $x = 0$ c) $y = x$ d) $y = -x$



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Ans a) (3,-1) (1,-5) (4,-7) b) (-3,1) (-1,5)(-4,7) c) (1,3) (5,1) (7,4) d) (-1,-3) (-5,-1) (-7,-4)

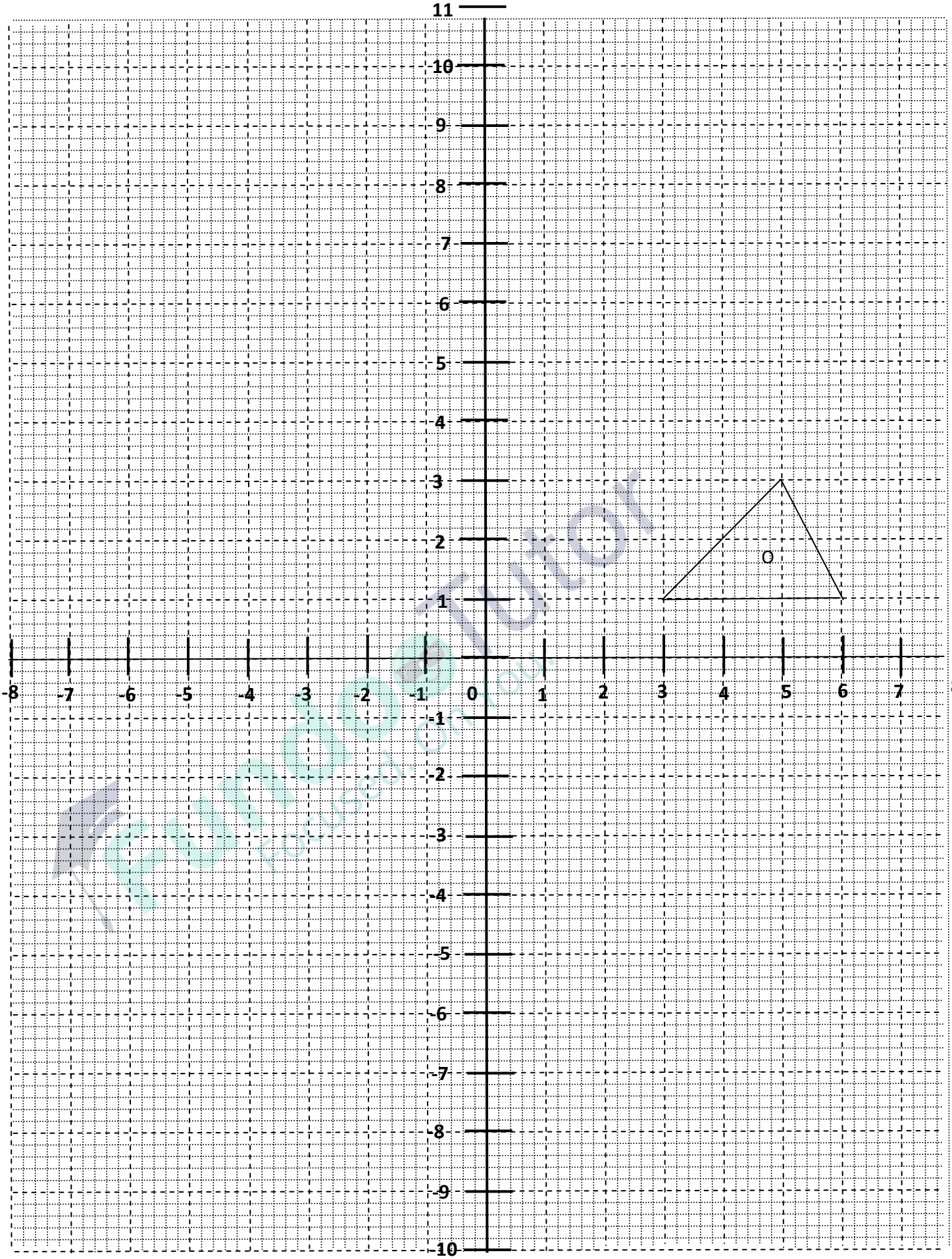
Q3) Reflect triangle O in the line **a)** $y = 5$ **b)** $x = -1$ **c)** $y = x + 2$ **d)** $y = -x - 2$ **e)** $x = 5$



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Ans a) (3,9) (6,9) (5,7) b) (-5,1) (-8,1)(-7,3) c) (-1,5) (-1,8) (1,7) d) (-3,-5) (-3,-8) (-5,-7) e)(7,1)(4,1)(5,3)

Q4) Reflect triangle O in the line **a)** $y = -2$ **b)** $x = 2$ **c)** $y = x - 3$ **d)** $y = -x + 2$ *e)* $y = 1$

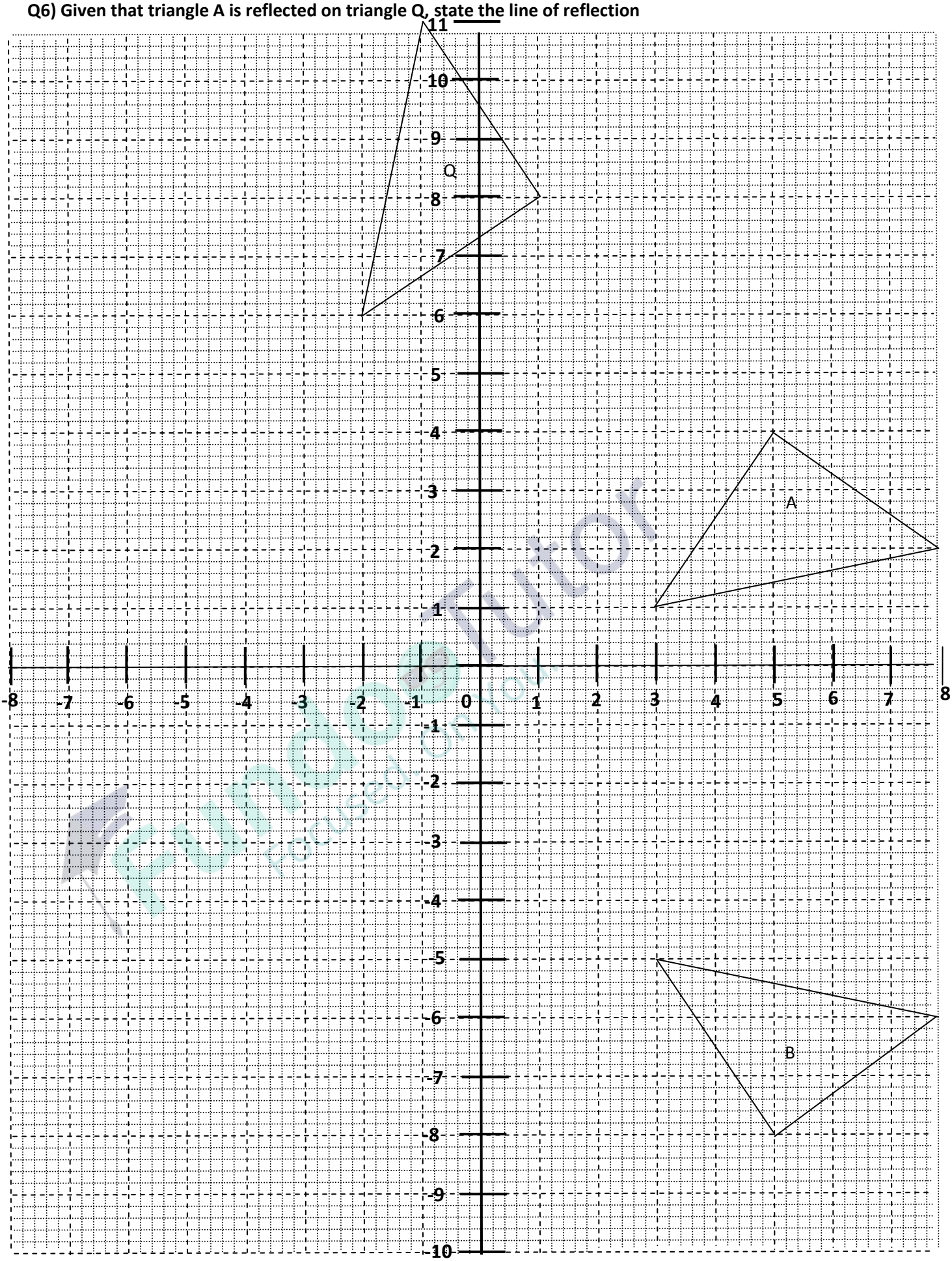


Ans a) (3, -5) (6, -5) (5, -7) b) (-2, 1) (1, 1) (-1, 3) c) (4, 0) (6, 2) 4, 3) d) (-1, 3) (1, 1) (1, -4) e) (3, 1) (6, 1) (5, -1)

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Q5) Given that triangle A is reflected on triangle B, state the line of reflection

Q6) Given that triangle A is reflected on triangle Q, state the line of reflection



Ans **Q5) $y = -2$** **Q6) $y = x + 3$**

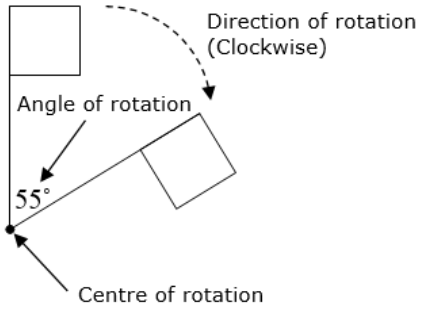
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2) Rotation (R) :-

A **rotation** is a transformation in which the object is rotated about a fixed point. The **direction of rotation** can be **clockwise** or **anticlockwise**.

The fixed point in which the rotation takes place is called the **center of rotation**. The amount of rotation made is called the **angle of rotation**.

Example:



(a) Rotating 90° , 180° , 270° about given centre using tracing paper

1) Place tracing paper over figure and trace the figure and trace small (1 to 2cm long) vertical and horizontal line around the centre



2) place pencil tip around the centre, to secure the tracing paper and rotate the page according to given angle

(e.g. for 90° clock wise rotation bring top corner of the vertical line to right corner of the horizontal line)

3) Copy the new location of the figure from tracing paper to the question paper below

(b) Rotating by other angles about given centre using tracing paper

[eg 30° anti clockwise rotation]

1) Draw small (1 to 2cm long) vertical and horizontal line around the centre



2) Make a line at 30° anticlock wise angle about the centre



3) Place tracing paper over figure and trace the figure and the vertical and horizontal line drawn around the centre

4) place pencil tip around the centre, to secure the tracing paper and rotate the page according to given angle

(in this case bring the right side of the horizontal line to the top of the line made at 30°)

5) Copy the new location of the figure from tracing paper to the question paper below

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(c) Finding the centre of rotation and angle of rotation from two given rotated figures

(i) Centre of rotation

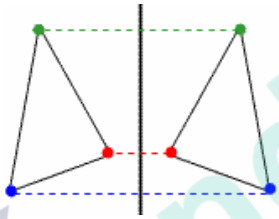
1) Place tracing paper over the figure and trace the original figure
2) Randomly choose a coordinate as the centre and place pencil tip on it and rotate the page towards the transformed figure
3) If the two figures overlap your chosen point is the centre , otherwise repeat procedure with another randomly chosen centre

(ii) Angle of rotation

1) Ones the centre has been found join the centre with any corner of the original figure with a straight line , again join the centre with the corresponding corner of the transformed figure.
2) Measure the angle formed with the D and state the direction of rotation (clock wise or anti clock wise)

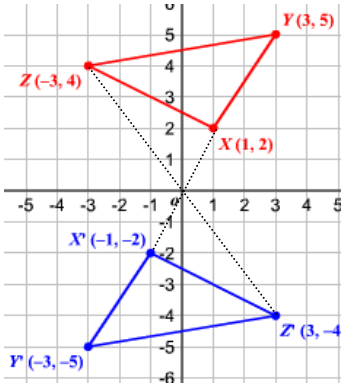
(e) Distinguishing between Rotation and Reflection

How to tell if two transformed shapes are reflection or rotation of each other? Simply join the corresponding points of the two shapes. If all lines are parallel it is a reflection , if even one line is not parallel to the rest it is a rotation.



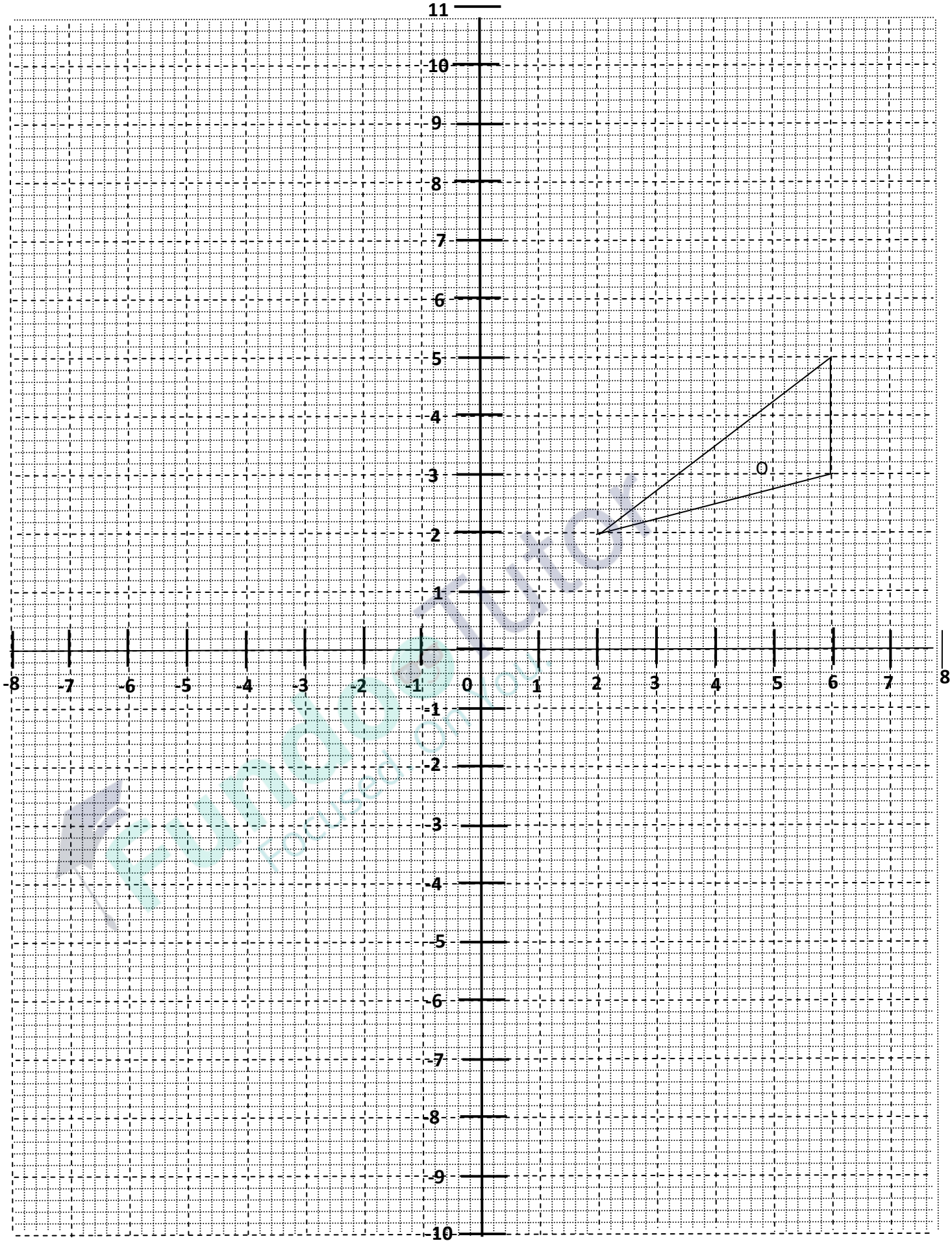
As lines are parallel hence reflection

As lines are not parallel, hence rotation



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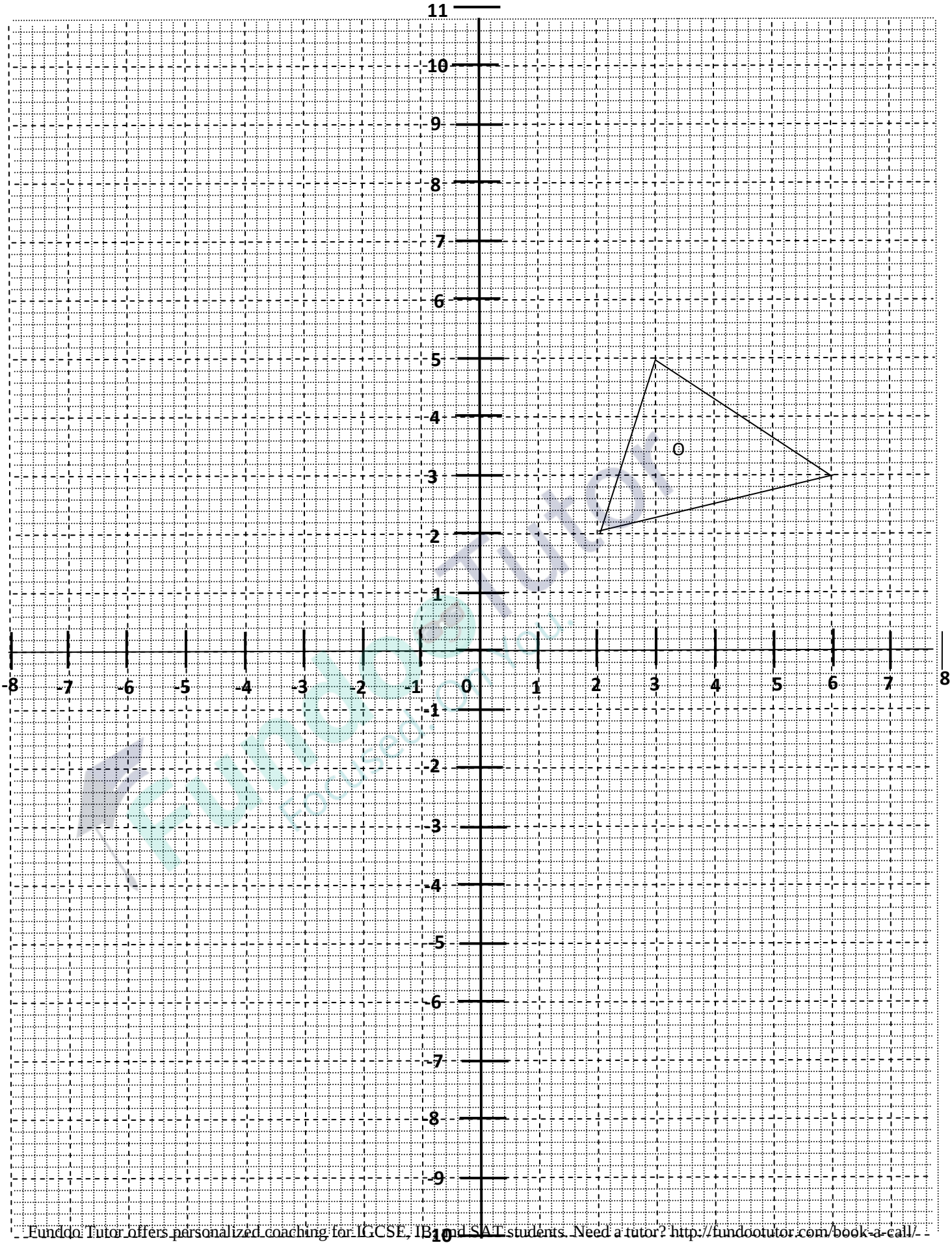
Q1) Rotate Triangle O about the origin at angle a)90° cw b) 90° ccw c) 180°



Ans a)(2,-2) (3,-6) (5,-6) b)(-2,2) (-3,6) (-5,6) c) (-2,-2) (-6,-3) (-6,-5)

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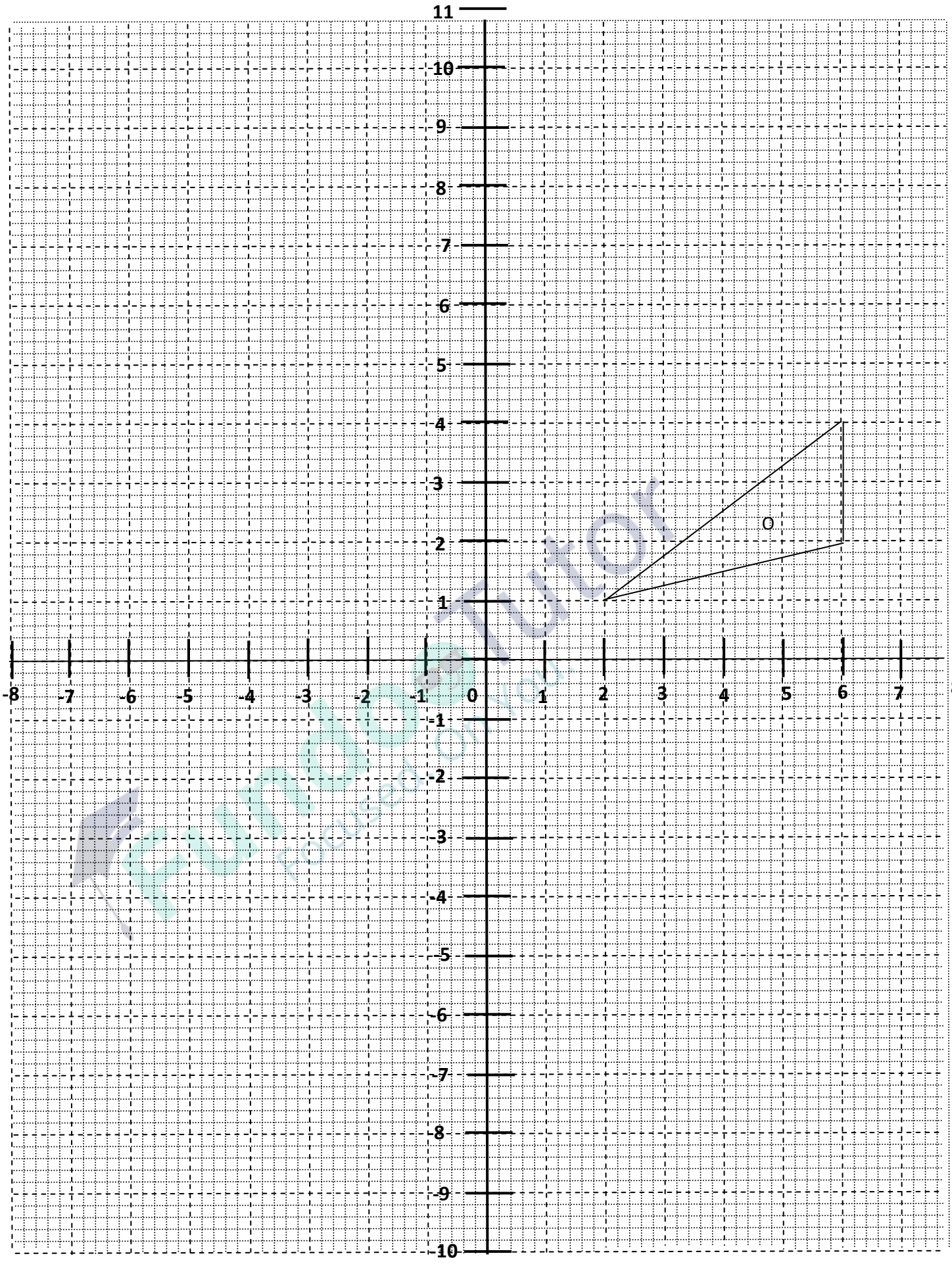
Q2) Rotate Triangle O about the origin at angle a)90° cw b) 90° ccw c) 180°



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Ans a)(2,-2) (3,-6) (5,-3) b) (-2,2) (-3,6) (-5,3) c) (-2,-2) (-6,-3) (-3,-5)

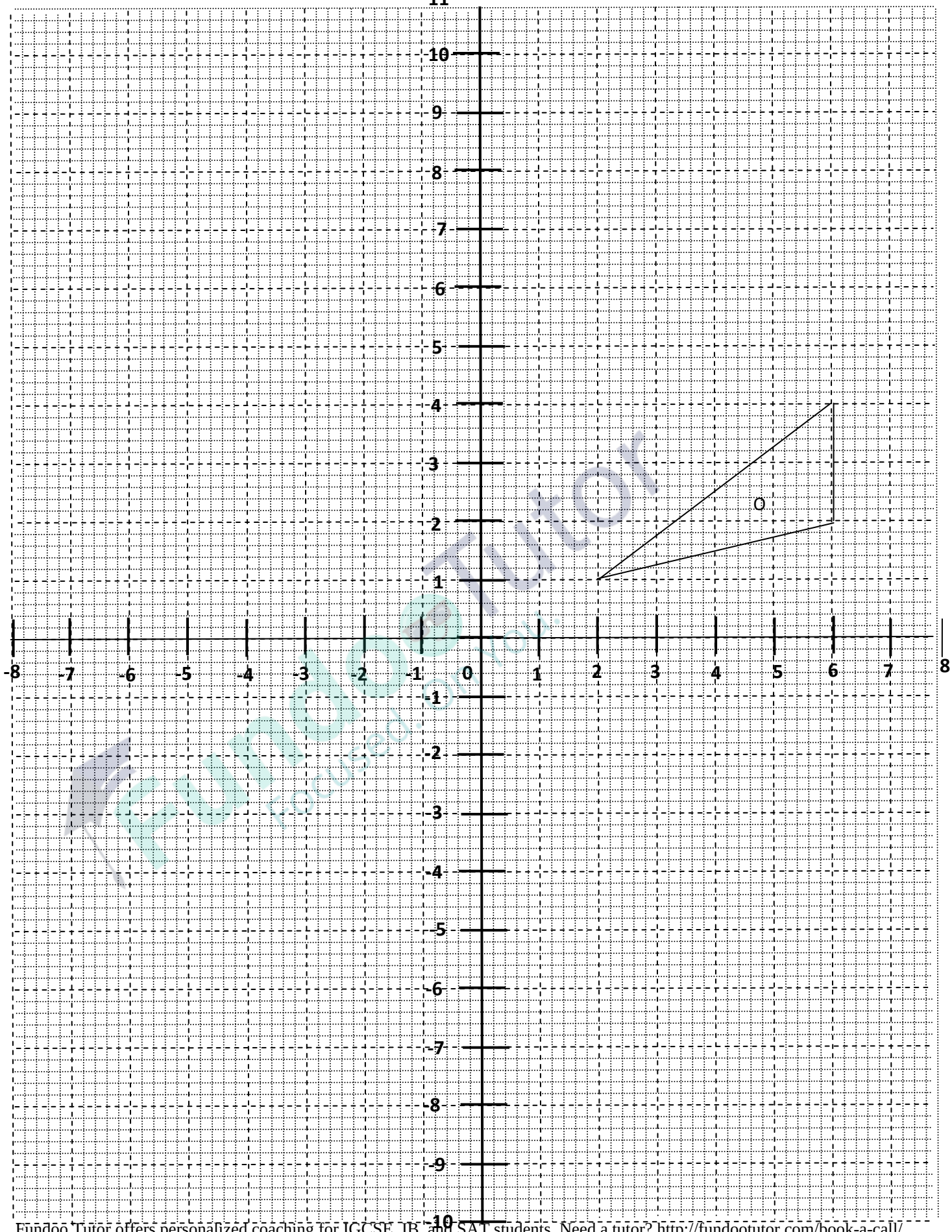
Q3) Rotate Triangle O about the centre (1,4) at angle a)90° cw b) 90° ccw c) 180°



Ans a) (-2,3) (-1,-1) (1,-1) b) (4,5) (3,9) (1,9) c) (0,7) (-4,4) (-4,6)

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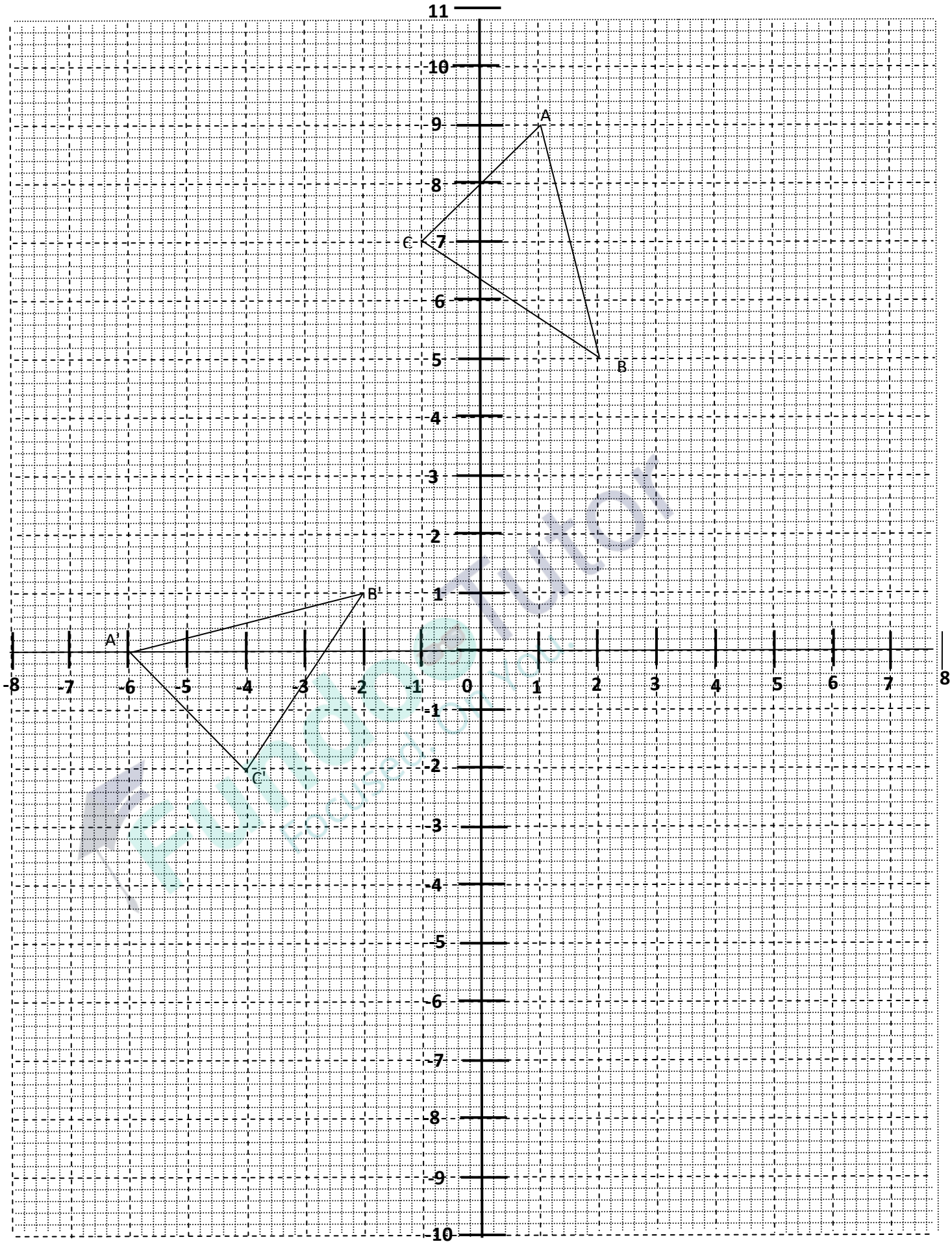
Q4) Rotate Triangle O about the centre (2,-1) at angle a)90° cw b) 90° ccw c) 180°



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Ans a)(4,-1) (5,-5) (7,-5) b) (0,-1) (-1,3) (-3,3) c) (2,-3) (-2,-4) (-2,-6)

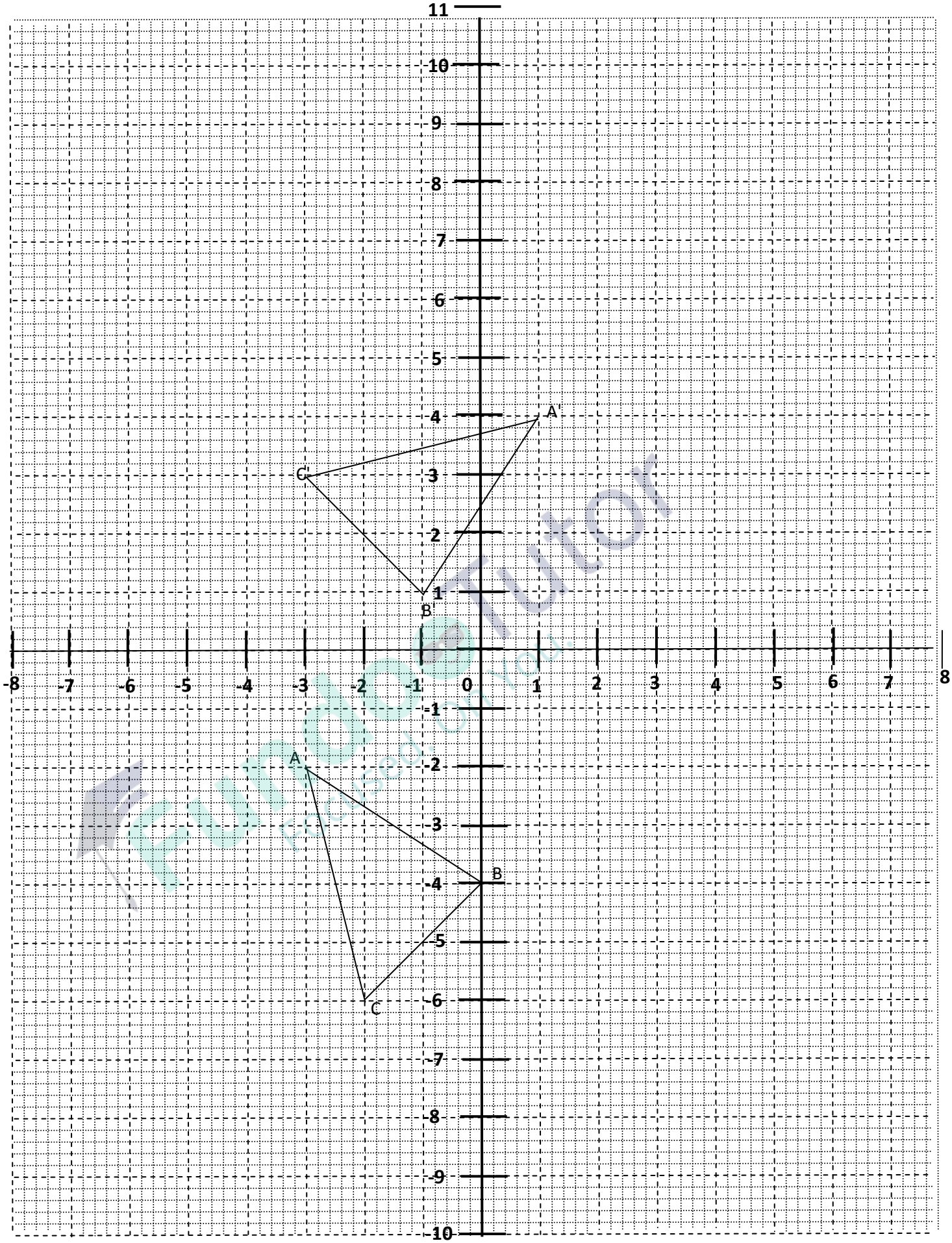
Q5) Triangle ABC is mapped onto triangle A'B'C' by a transformation. Describe this transformation completely.



Ans It is a rotation with centre (2,1) and angle 90° ccw or 270° cw

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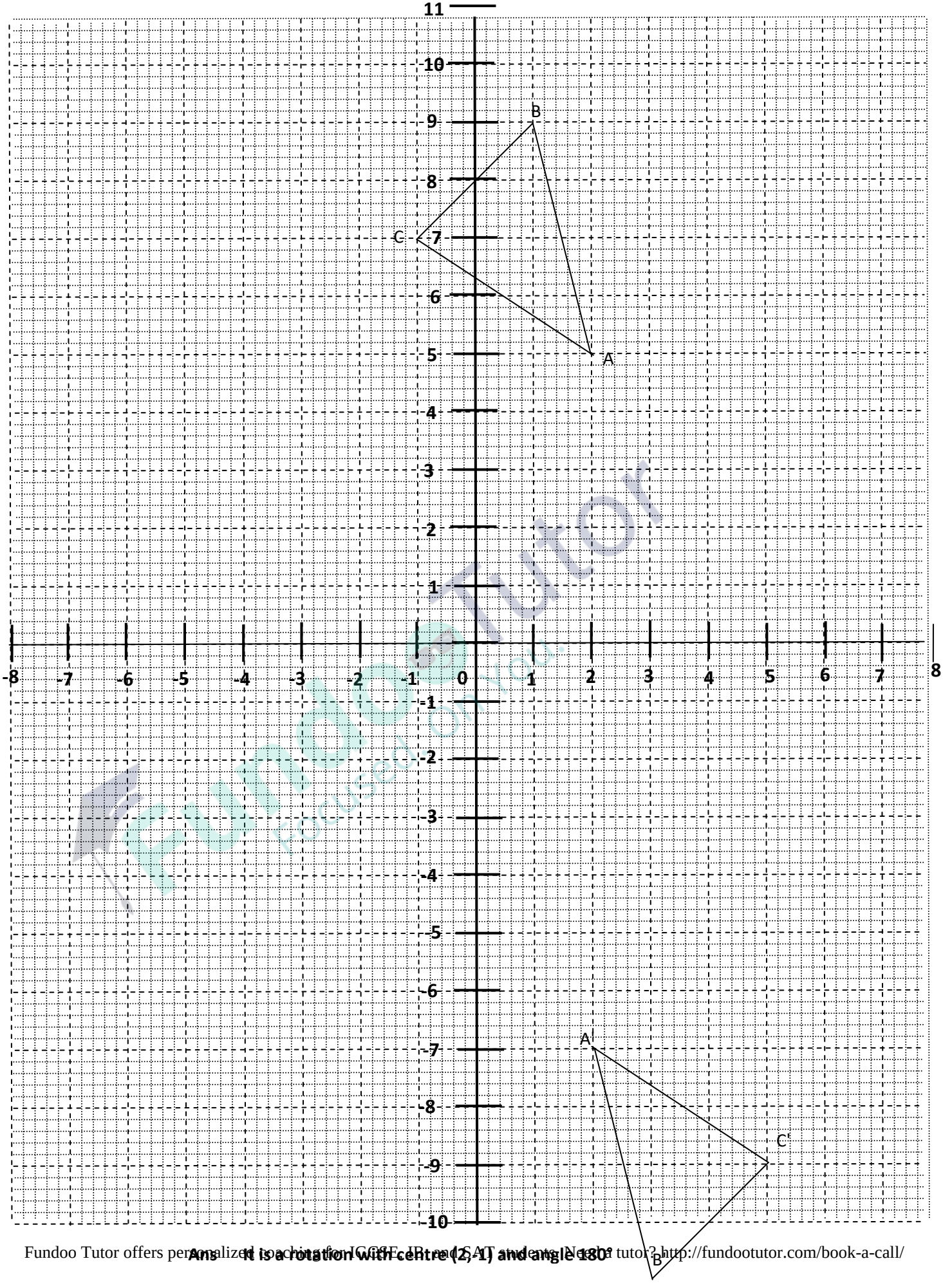
Q6) Triangle ABC is mapped onto triangle A'B'C' by a transformation. Describe this transformation completely.



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Ans It is a rotation with centre (2,-1) and angle 90° cw or 270° ccw

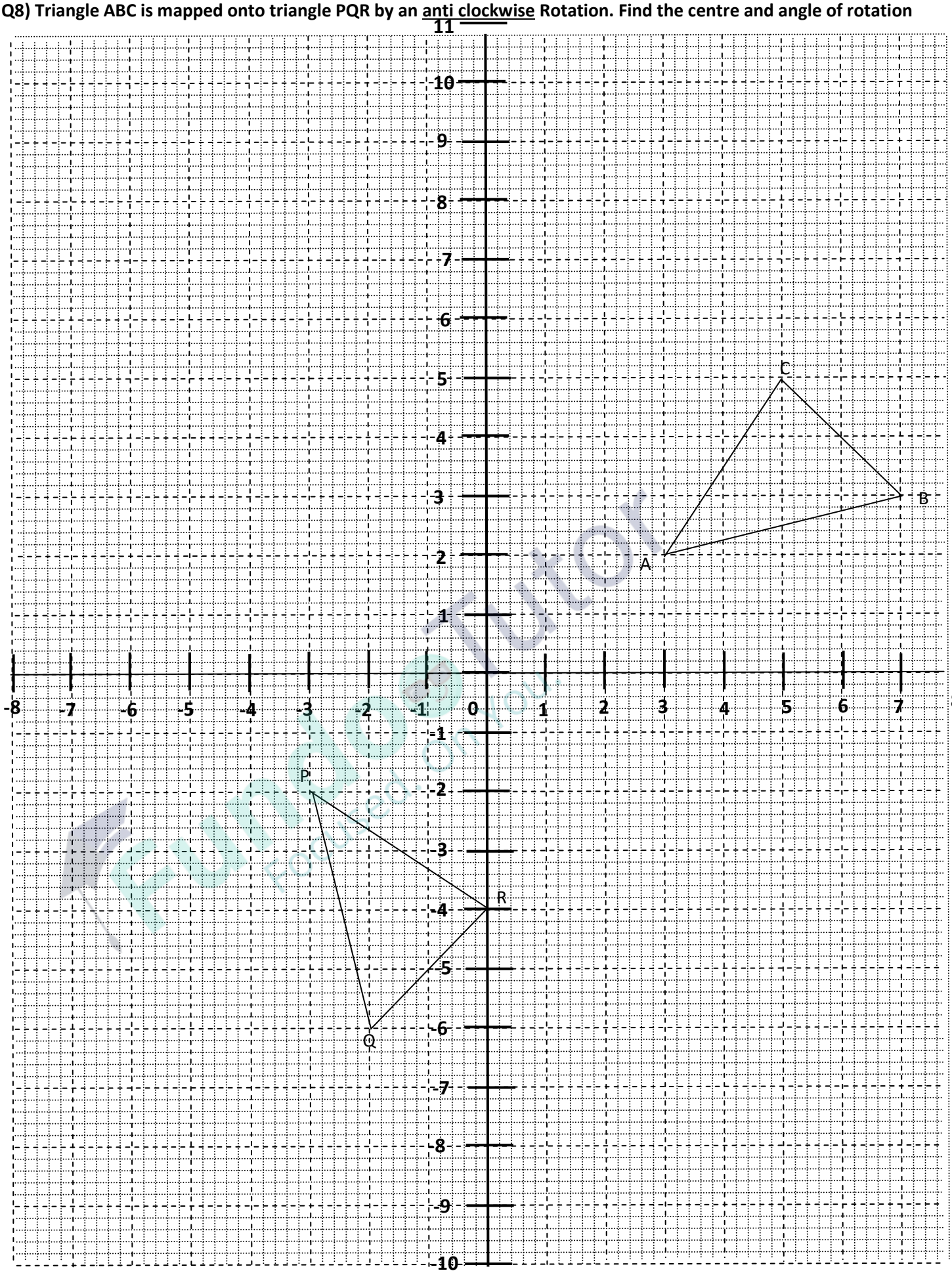
Q7) Triangle ABC is mapped onto triangle A'B'C' by a transformation. Describe this transformation completely.



Ans: It is a rotation with centre (2,-1) and angle 180°

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Q8) Triangle ABC is mapped onto triangle PQR by an <u>anti clockwise</u> Rotation. Find the centre and angle of rotation



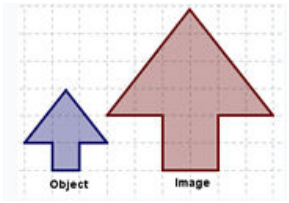
Ans centre (-2,3) and angle 270

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3) Enlargement (E) :-, It is a kind of transformation that changes the size of an object. The image created is similar to the object (same shape , different size).

Every enlargement has a **centre** and **scale factor**.

The scale factor is how many times larger than the object the image is.



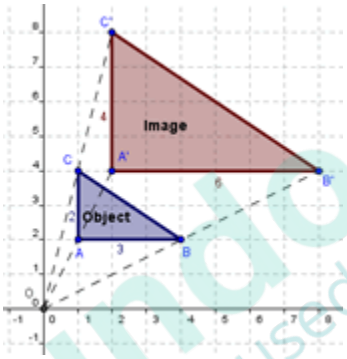
For every enlargement, a **scale factor**, k, must be specified:

$$k = \frac{length\ of\ side\ on\ image}{length\ of\ same\ side\ on\ object} \quad or$$

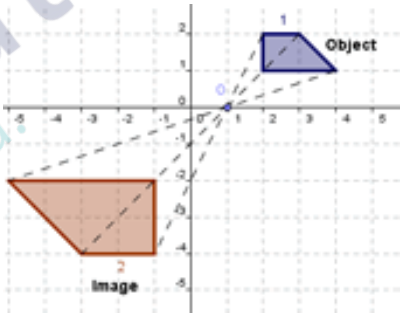
$$k = \frac{distance\ from\ centre\ of\ enlargement\ to\ point\ on\ image}{distance\ from\ centre\ of\ enlargement\ to\ same\ point\ on\ object}$$

Despite the name enlargement, it includes making objects smaller because the shape can be bigger or smaller according to the scale factor, k.

- If k>1, then the image is larger than the object and in same side of centre of enlargement.
- If k = 1, then the image and the object is the same size and in same side of centre of enlargement.
- If 0<k<1, then the image is smaller than the object and in same side of centre of enlargement.
- If k is negative the image is in the opposite side of centre of enlargement.



Scale factor 2



Scale factor -2

Also note

Area of image = k^2 x Area of object (derived from $\frac{A1}{A2} = (\frac{l1}{l2})^2$ of similar figures)

a) Enlargement when centre is (0,0)

Simply multiply each coordinate of the object with the scale factor to get each coordinate of the image

b) Enlargement when centre is not (0,0)

Step 1: Note column vector from the centre to each vertex (corner)of the figure

Step 2: Multiply each column vector with the given scale factor

Step 3: Redraw each point using the new column vector (from the centre)

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d) Finding centre of enlargement and scale factor of two enlarged shapes

Step 1: Join corresponding points of object and image (i.e. points A to A' , B to B' and C to C'). The point of intersection is the **centre of enlargement**.

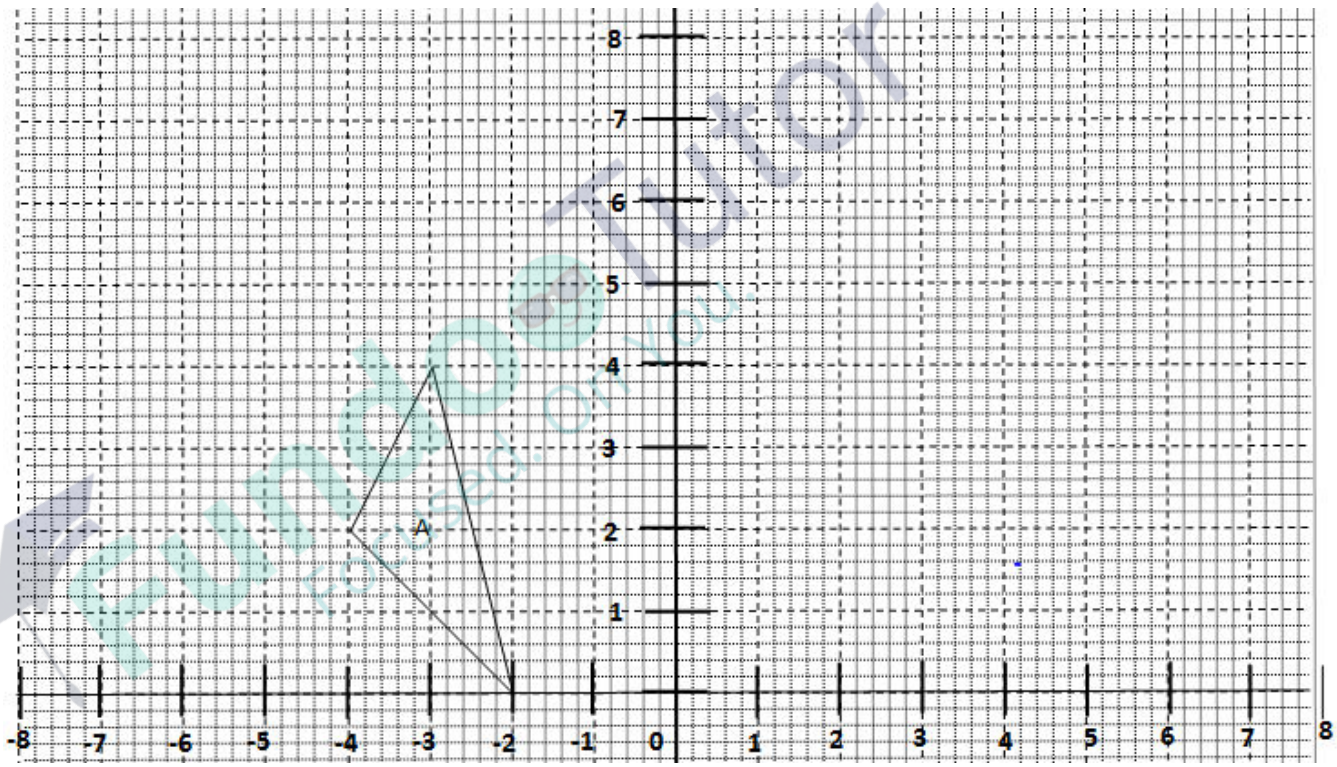
Step 2: Find **scale factor** using $k = \frac{length\ of\ side\ on\ image}{length\ of\ same\ \ side\ on\ object}$

or $k = \frac{new\ length}{corresponding\ old\ length}$

Q1) Map Triangle A on

a)Triangle P by an enlargement with centre(0,0) and scale factor 2

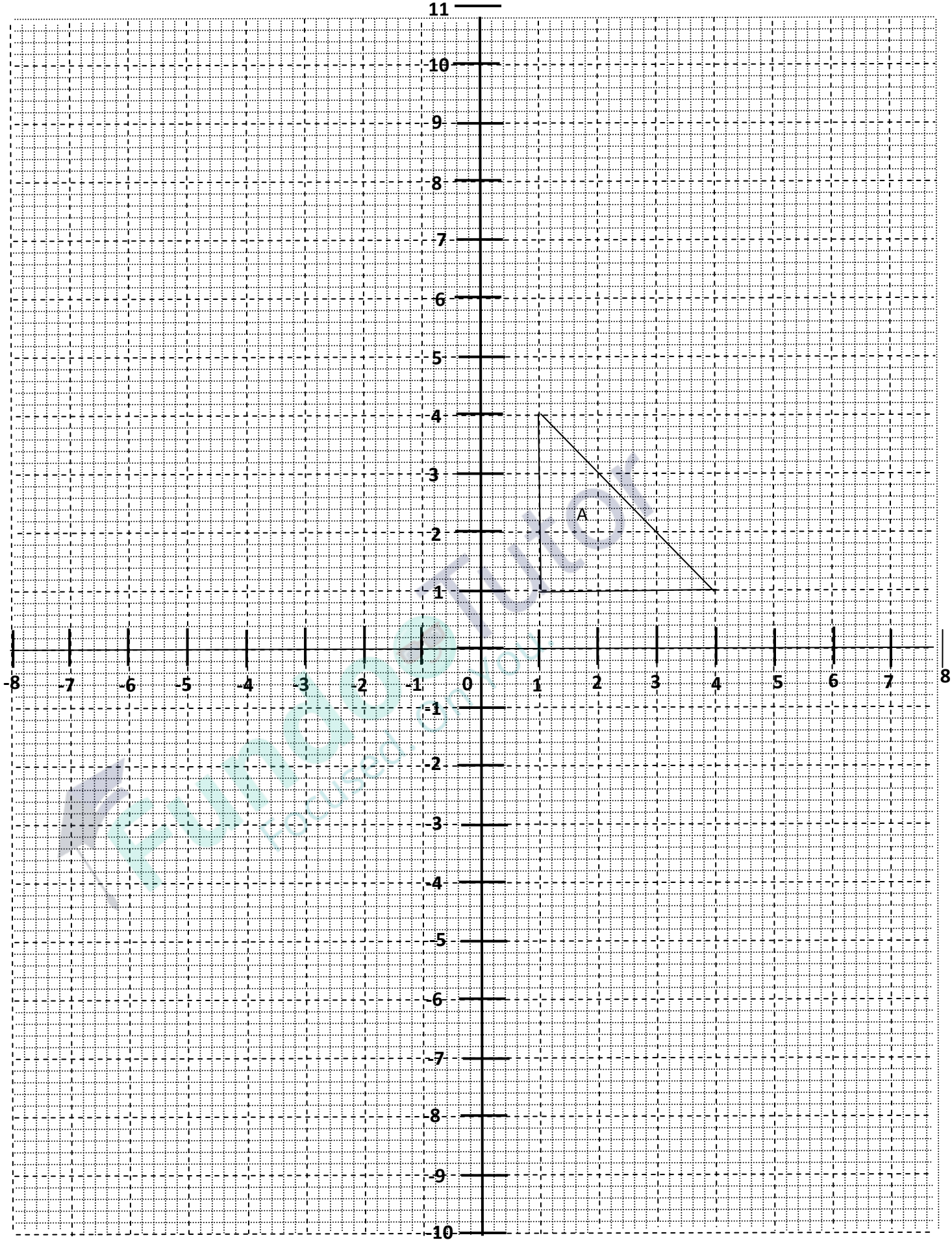
b) Triangle Q by an enlargement with centre(0,0) and scale factor 1/2



Ans P(-4,0) (-8,4) (-6,8) Q(-1,0) (-2,1) (-1.5,2)

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Q2) Map Triangle A by an enlargement with centre (0,0) and scale factor a) 2 b)1/2 c)-2 d)-1/2

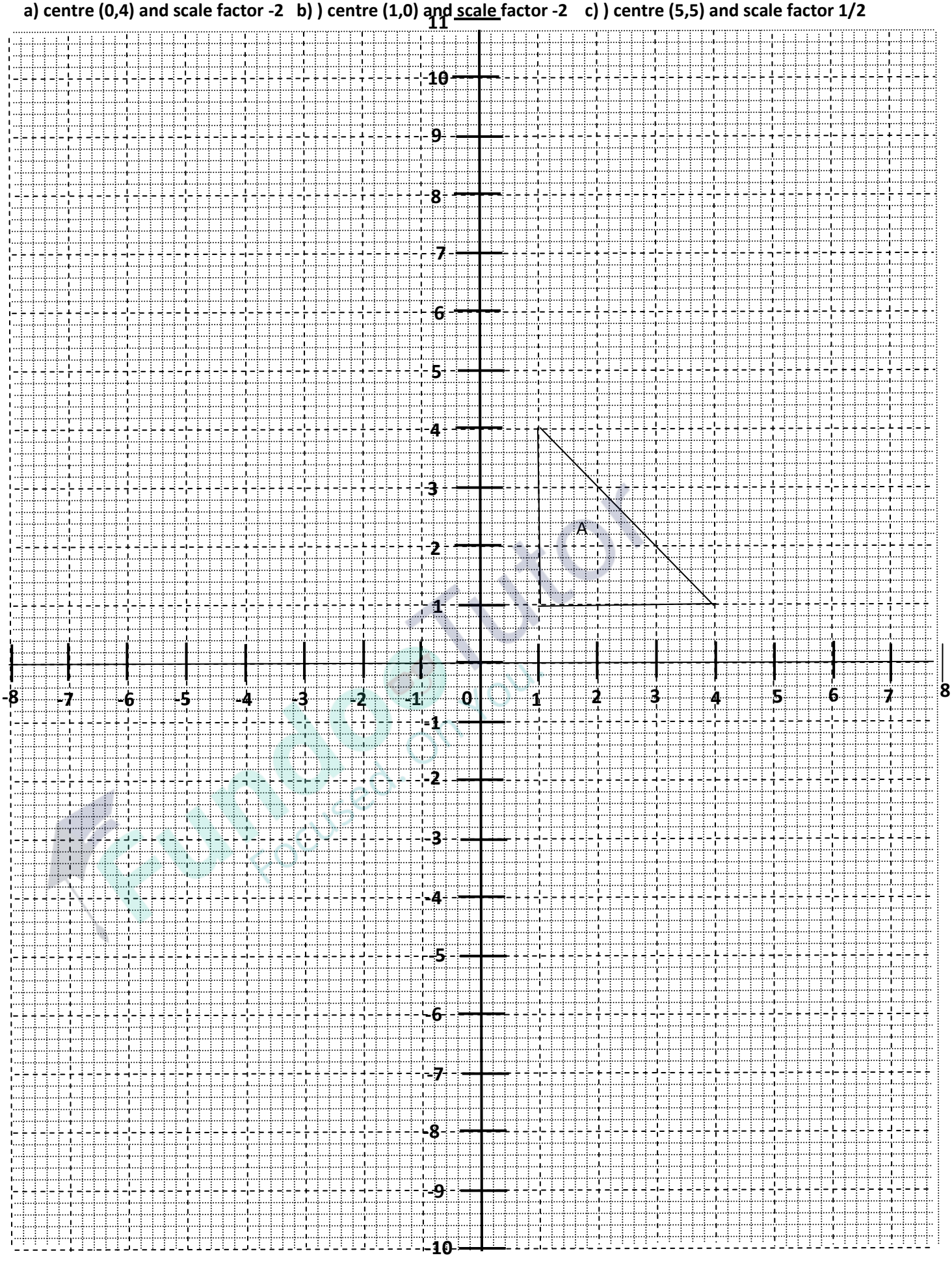


Ans a)(2,2)(8,2)(2,8) b)(0.5,0.5)(2,0.5)(0.5,2) c) (-2,-2)(-8,-2)(-2,-8) d) (-0.5,0.5)(-2,0.5)(-0.5,-2)

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Q3) Map Triangle A by an enlargement with

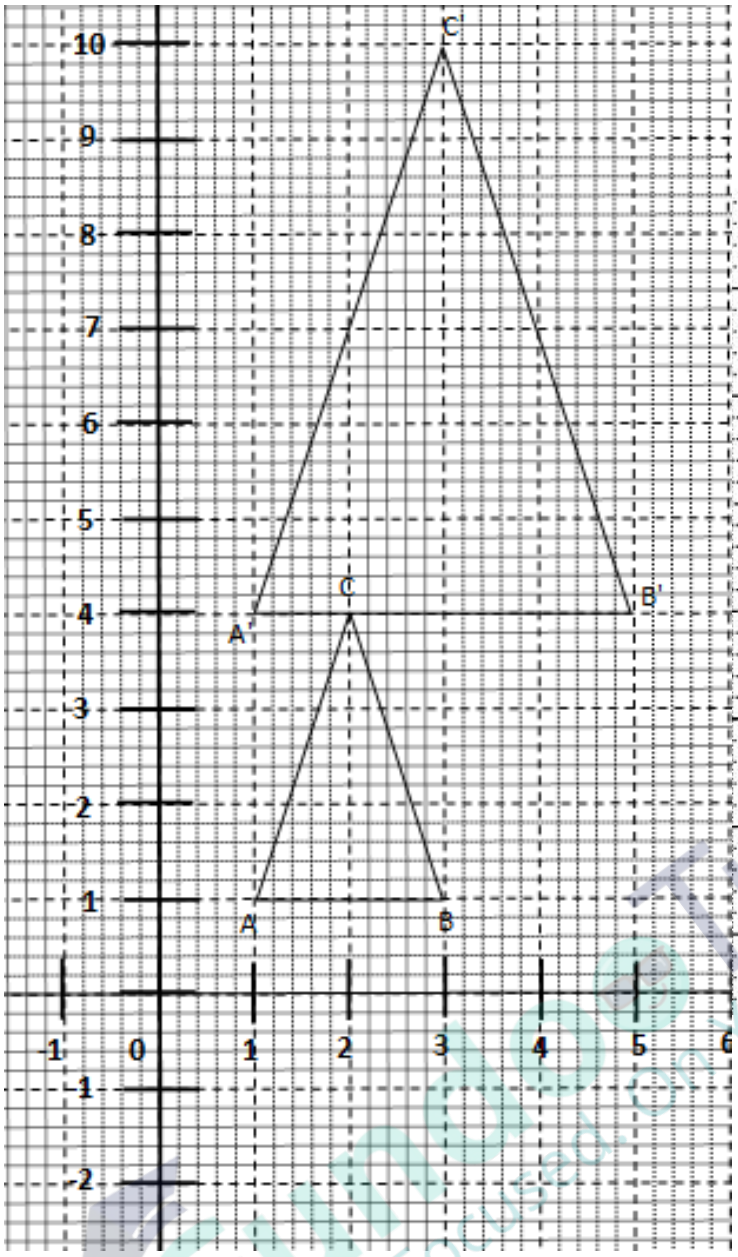
a) centre (0,4) and scale factor -2 b)) centre (1,0) and scale factor -2 c)) centre (5,5) and scale factor 1/2



Ans a)(-2,4) (-2,10)(-8,10) b)(1,-2)(1,-8)(-5,-2) c) (3,3) (4.5,3)(3,4.5)

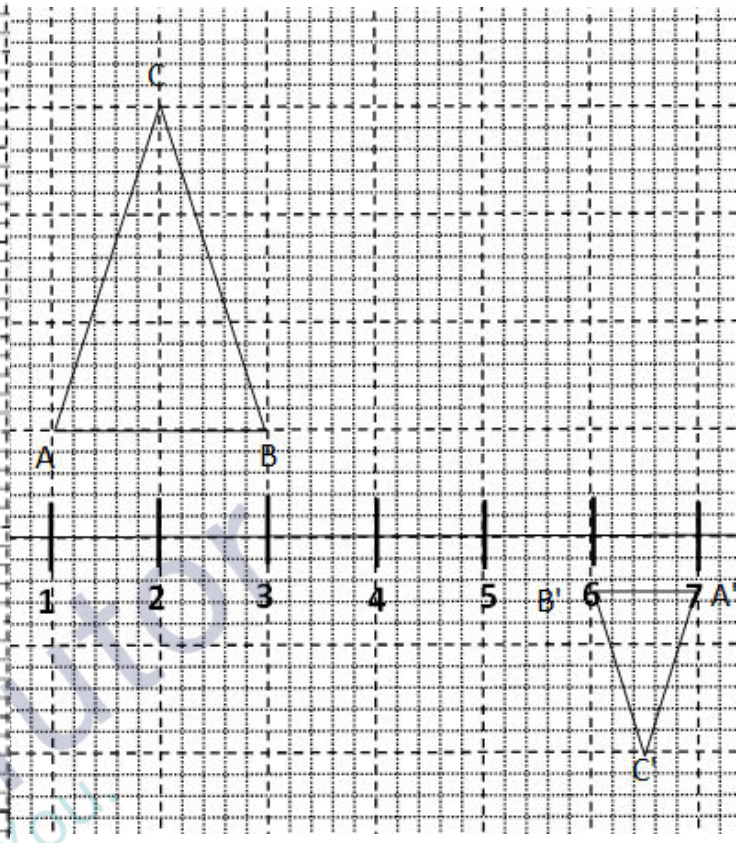
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Q4) Given △ABC is transformed to △A'B'C'. Completely describe this transformation.



Ans It is an enlargement with centre (1,-2) and scale factor 2

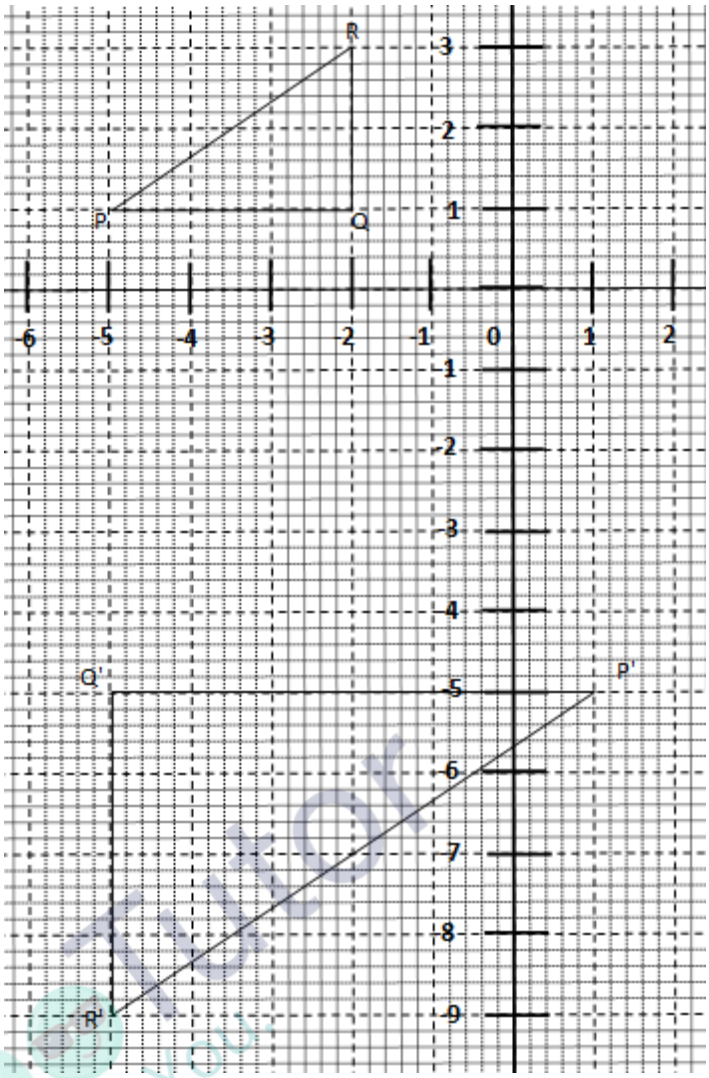
Q5) Given △ABC is transformed to △A'B'C'. Completely describe this transformation.



Ans It is an enlargement with centre (5,0) and scale factor -1/2

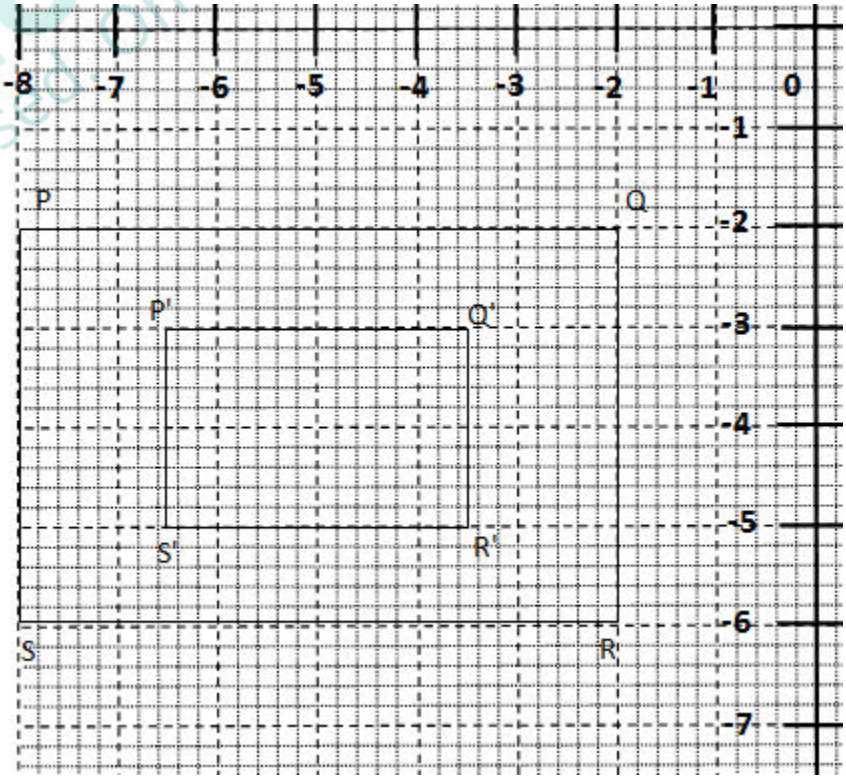
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Q6) Given △ PQR is transformed to P'Q'R'. Completely describe this transformation.



Ans It is an enlargement with centre (-3,-1) and scale factor -2

Q7) Given rectangle PQRS is transformed to P'Q'R'S'. Completely describe this transformation.



Ans It is an enlargement with centre (-5,-4) and scale factor 1/2

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4) Translation (T) :- a **translation** is a geometric transformation that moves every point of a figure by the same distance according to a given column vector. The size, the shape and the orientation of the image are the same as that of the original object.

a) Column Vector $\begin{pmatrix} x \\ y \end{pmatrix}$

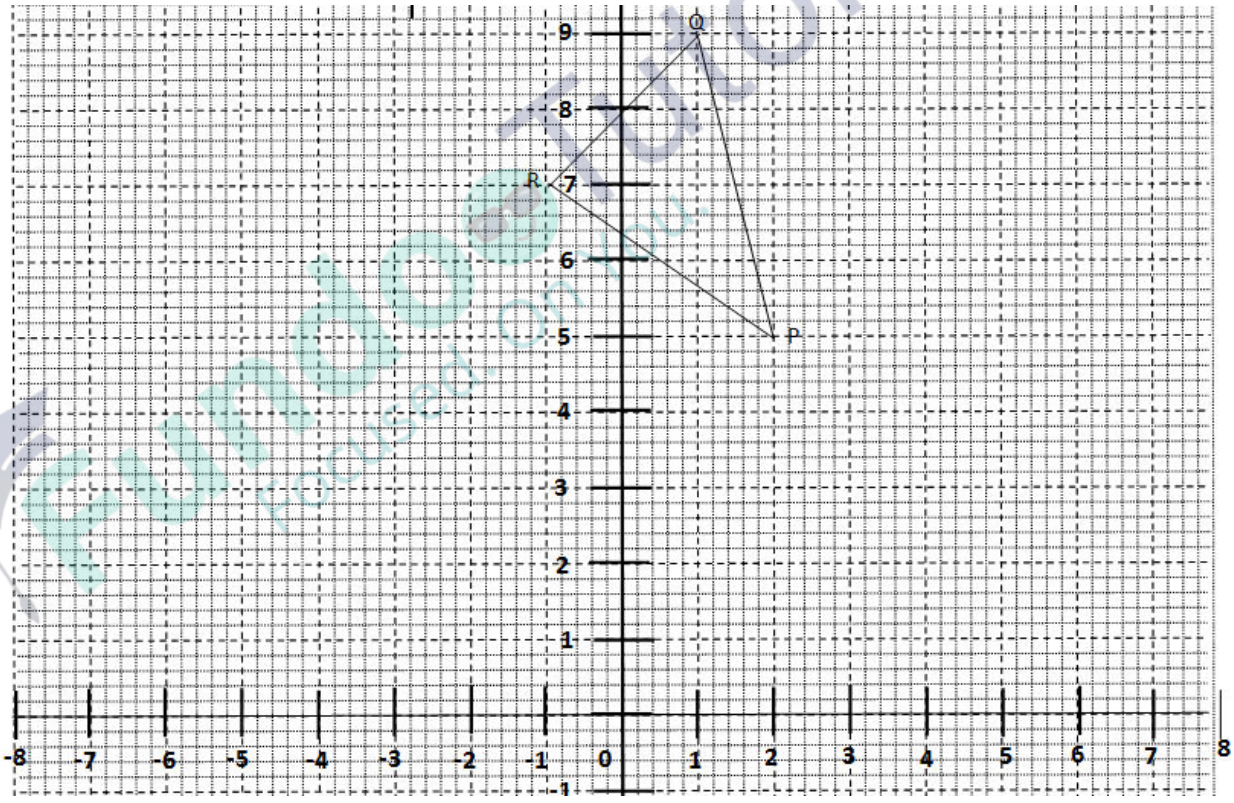
The object is moved according to given column vector.

When x is positive each point is moved x units right and when x is negative each point is moved x units left.

Similarly when y is positive every point is moved y units up and when y is negative every point is moved y units down.

e.g. $\begin{pmatrix} 3 \\ -2 \end{pmatrix}$ means move every point of the object 3 units right and 2 units down to get the image coordinates.

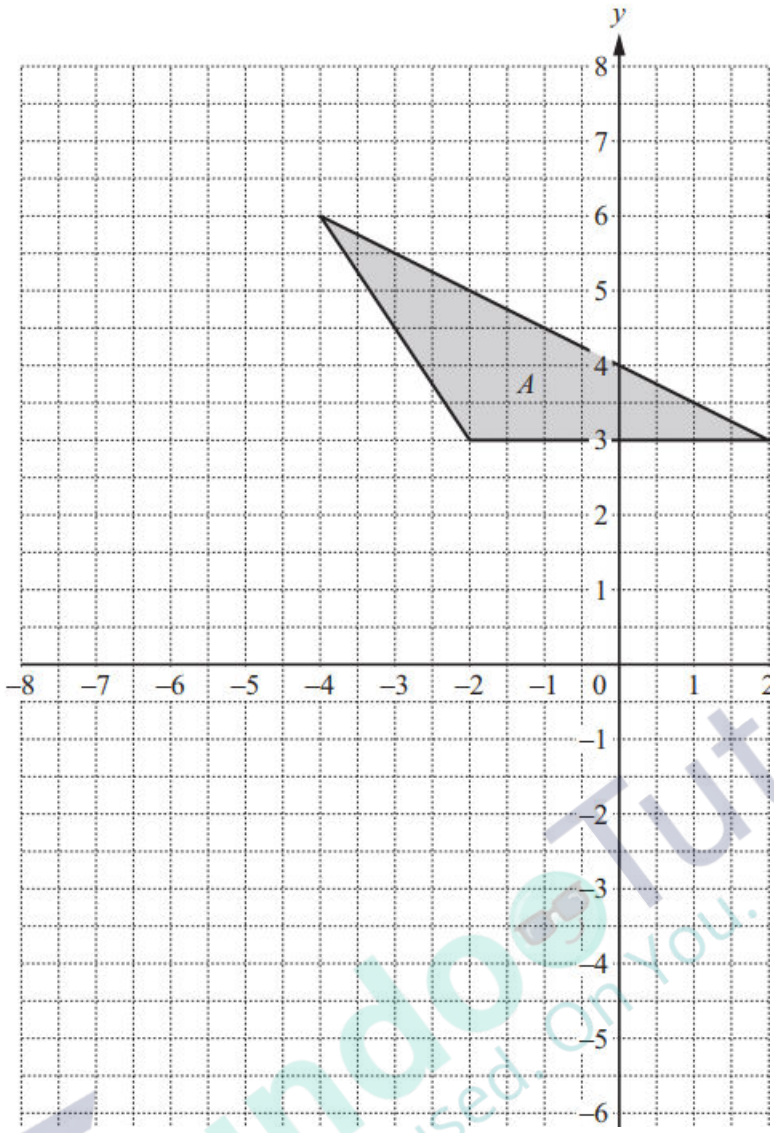
Q1) Triangle PQR is mapped on Triangle P'Q'R' by a translation of given matrix. Draw triangle P'Q'R' a) $\begin{pmatrix} 3 \\ -2 \end{pmatrix}$ **b)** $\begin{pmatrix} -4 \\ -5 \end{pmatrix}$



Ans a)(5,3)(2,5)(4,7) b)(-2,0) (-5,2) (-3,4)

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Q2) On the Grid draw the image of triangle A after translation by the vector $\begin{pmatrix} -1 \\ -5 \end{pmatrix}$

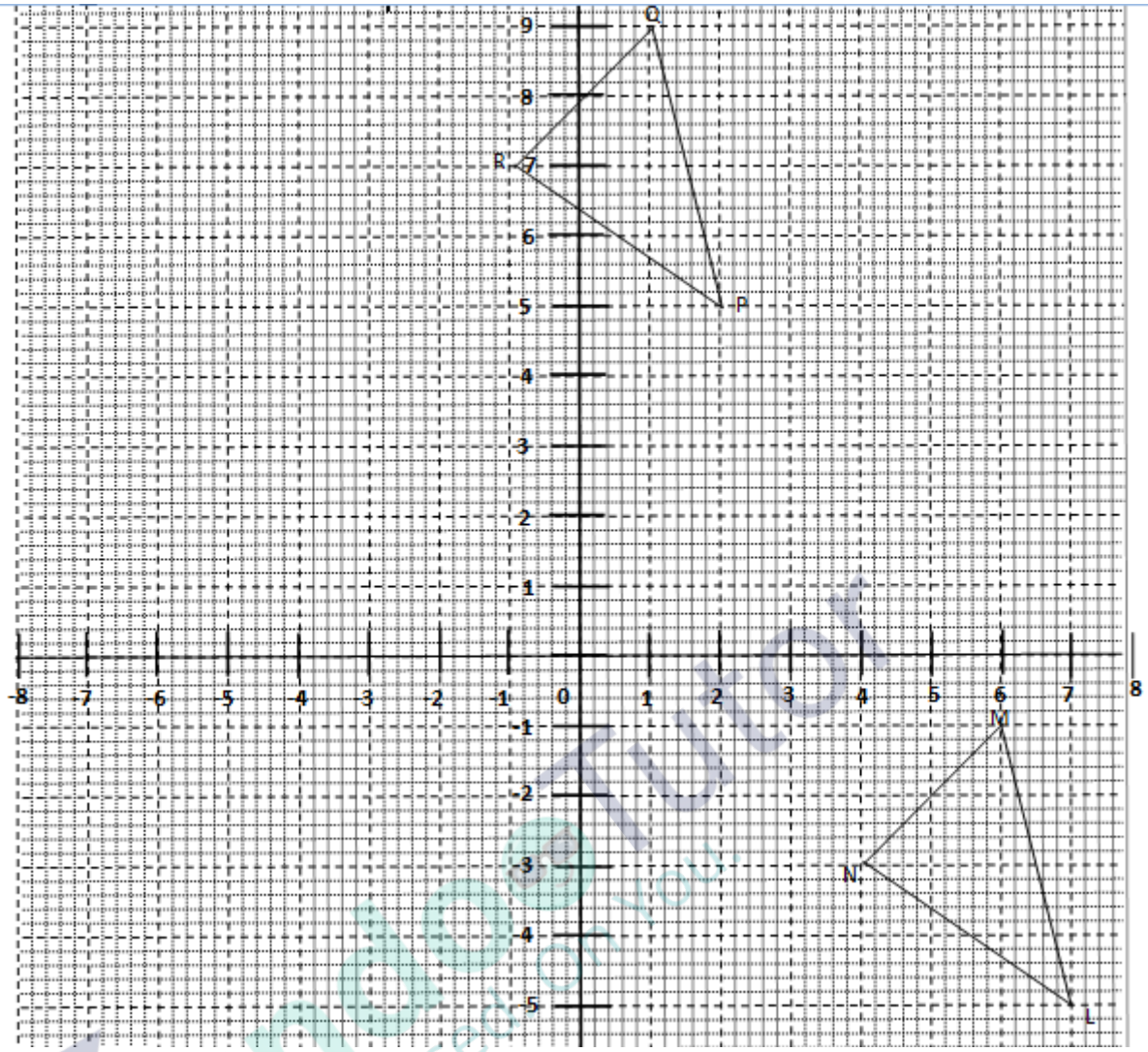


Ans a) Image at (-5,1),(-3,-2),(1,-2)

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Q3) a) Triangle PQR is mapped on Triangle LMN by a transformation, describe this transformation completely.

b) Given L(-4,4) and M(-2,0). Given that a Translation 'T' maps L onto M. Find image of triangle PQR under translation T.



Ans a) Translation by vector $\begin{pmatrix} 5 \\ -10 \end{pmatrix}$ b) (1,3) (3,5) (4,1)

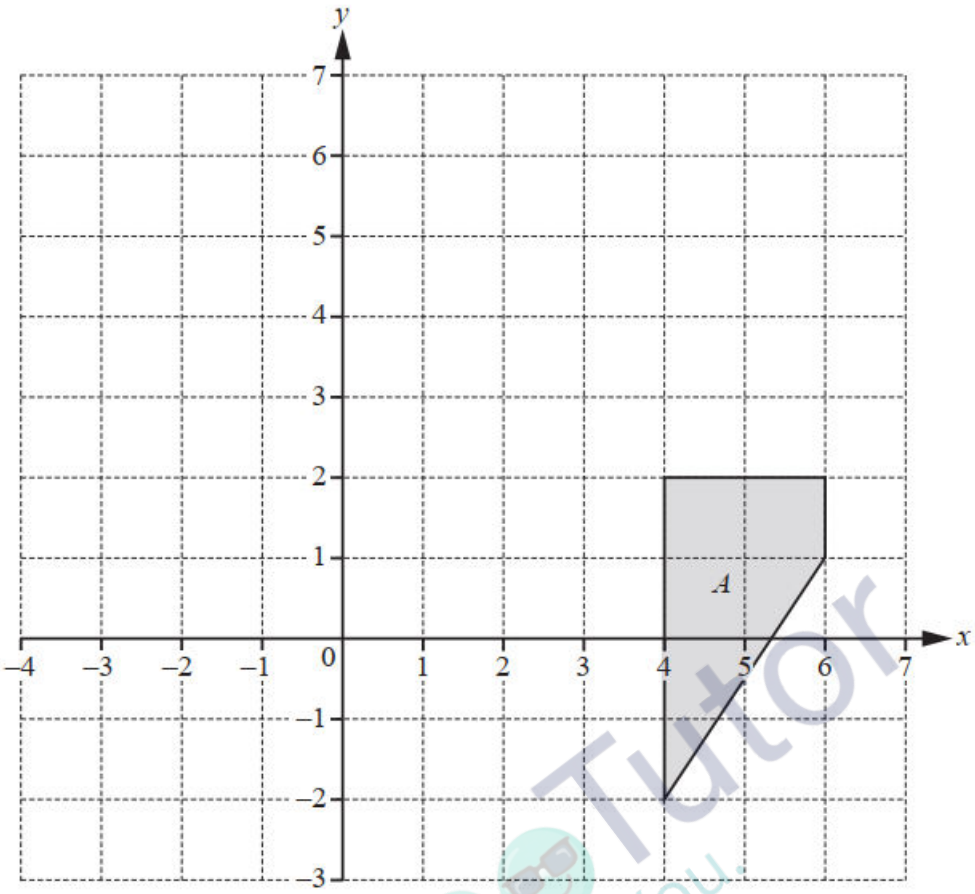
Past Paper Question

Q) T(X) is the image of the shape X after translation by the vector $\begin{pmatrix} -1 \\ 3 \end{pmatrix}$

M(Y) is the image of the shape Y after reflection in the line $x = 2$.

On the grid, draw MT(A), the image of shape A after the transformation MT. [3]

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0580/22/F/M/17 Q11)(-1,4) (-1,5) (1,5) (1,1)

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